The inhomogeneity of grid turbulence

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Summary

The intensity of turbulent motion in the wake of square mesh grids of bars has been examined experimentally. The measurements show that there are considerable departures from homogeneity over surfaces normal to the mean stream. Large variations of intensity have been observed at a distance of 80 meshlengths from the grid, where they are diminishing slowly, if at all. This inhomogeneity is believed to be the cause of the discrepancies between various sets of measurements of energy decay in the 'initial period'. There is also a possibility of error in other measurements made in grid turbulence and analysed on the assumption of homogeneity. Observations with a grid of 2 in. mesh show that $\overline{v^2}$ is significantly less than $\overline{u^2}$, which is consistent with results from other laboratories.

Introduction

The simplest type of three-dimensional turbulence that can be imagined is isotropic homogeneous turbulence, in which the statistical properties of the motion are unchanged by rotation or reflection of the coordinate axes. These restrictions have permitted much greater theoretical progress than has been possible with more complicated forms (Batchelor 1953). The closest practical approximation to this appears to be the turbulence in the wake of a square-mesh grid of bars; this suffers from the disadvantage that the intensity of the turbulence decays with time and hence with distance from the grid, but this departure from spatial homogeneity in the direction of the mean flow is found to be small over lengths comparable to the linear scale of the turbulence. Since Taylor (1935) first suggested the use of grids for generating turbulence, it has been common practice to assume that this 'grid turbulence' is homogeneous and isotropic in this approximate sense, and it has been used almost exclusively in the experimental studies which support the theory of isotropic turbulence.

It had recently become clear from unpublished work by Corrsin at Johns Hopkins University and by Wyatt at Manchester that such turbulence is not accurately isotropic, and we therefore attempted to determine the ratio of the intensities of the down-stream and cross-stream components of turbulent velocity in the small wind tunnel in the Cavendish Laboratory.

By measuring this ratio behind several grids and over a wide range of turbulent intensities, it was hoped to establish standards for the calibration of hot wire anemometers.

During the investigation it was found that the turbulence was not only anisotropic but also inhomogeneous. This lack of homogeneity has previously been recognized by workers using very fine grids of mesh length $\frac{1}{4}$ in. or less (Batchelor & Townsend 1948; Batchelor & Stewart 1950), but it does not seem to have attracted attention in connection with the larger grids. We therefore turned our attention to an examination of the degree of inhomogeneity of the turbulence, this being an even more serious departure from the assumed conditions than lack of isotropy.

Experimental arrangement

The measurements were made in a wind tunnel with a working section 15 in. square and 8 ft. long. The turbulence was produced by passing the air through biplane grids of circular cylinders of mesh to diameter ratio 5·33. Each grid is made from two plane arrays of parallel, equally spaced, circular cylinders. The two arrays touch each other and have the directions of their cylinder axes at right angles to each other. Five grids have been used and it is convenient to designate them by their mesh lengths, which are $\frac{1}{4}$ in., $\frac{1}{2}$ in., 1 in., and (in two cases) 2 in. They are geometrically similar, except that the intersections of the $\frac{1}{2}$ in. grid are soldered, and the mesh to diameter ratio of the 1 in. grid is slightly greater than 5·33. The wires of the $\frac{1}{4}$ in. grid are under tension. One of the two 2 in. grids, which was kindly lent to us by Professor S. Corrsin of the Johns Hopkins University, is made of wood with small nails at the intersections; the other 2 in. grid and the 1 in. grid are made of brass rods with no mechanical connection at the intersections.

The mean stream velocity will be designated by U, the intensity of the component of turbulent velocity in this direction by $\overline{u^2}$, and the intensities of the turbulent velocity components parallel to the two sets of bars in the grid by $\overline{v^2}$ and $\overline{w^2}$. Wind speeds were either 6·3 or 11 m/sec.

The hot wire anemometers were of platinum-cored wollaston wire with a diameter of 0·00025 cm after etching. A single wire normal to the mean stream was used for most determinations of $\overline{u^2}$ but, in cases where two components were measured, an x-wire was used for both. The wire length was usually about 0·05 cm and never exceeded 0·1 cm for either single or x-wires. The wires were placed in a Wheatstone bridge and the unbalance amplified with appropriate compensation for the lag caused by the thermal capacity of the wire. The output was fed to a vacuum thermojunction supplying a millivoltmeter. In the case of a single hot wire normal to the mean stream, this arrangement yields $C\overline{u^2}$, where the calibration 'constant' C is a function of the dimensions of the wire, the mean velocity, the heating current, the gain of the amplifier, and the amount and distribution of dirt on the wire. To obtain the intensity relative to that at some reference point in the flow (or in some other flow) simply requires a reading of the mean square output at the reference point enough often to determine any drift in wire sensitivity. This is what has been done in cases where the data are given in 'arbitrary units'; the probable error is then about 1%.

Inhomogeneity of the downstream component

Figure 1 shows the spatial distribution of intensity of the downstream component of velocity near the axis of the tunnel behind the $\frac{1}{2}$ in. grid. The diagram represents a plane normal to the mean flow, and we have drawn lines of constant $\overline{u^2}$ covering an area approximately 6 in. square, 80 mesh-lengths downstream from the grid. The extreme variation of intensity in this example is about 30% of the mean, and the maximum gradient is about 20% in $\frac{1}{2}$ in. No cross-stream variations of mean velocity were found with a pitot tube and manometer, which was capable of detecting differences of 0·5%.

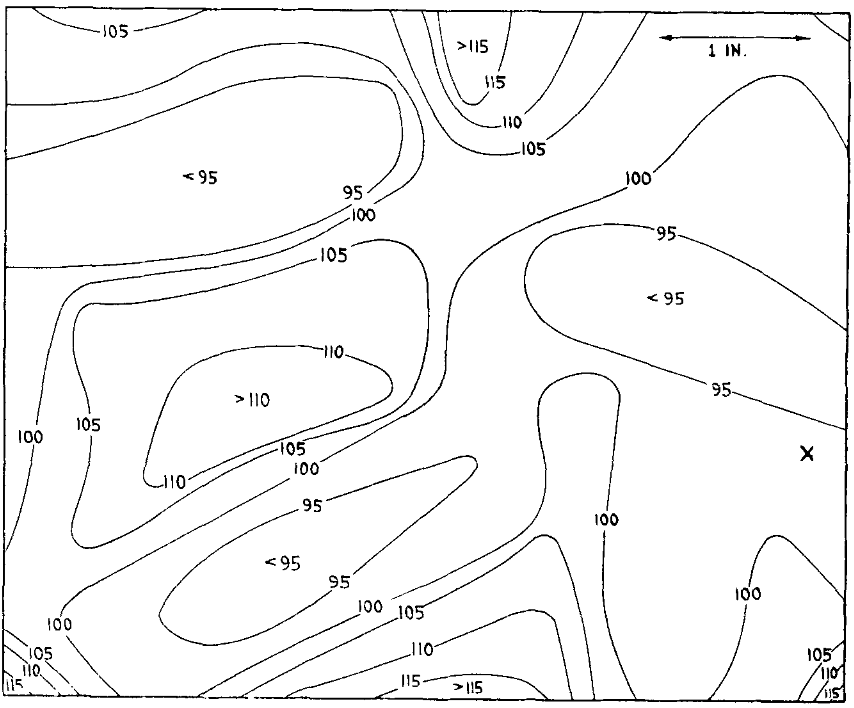

Figure 1. Distribution of $\overline{u^2}$ (in arbitrary units) over a plane normal to the mean flow and at 80 mesh-lengths downstream from the $\frac{1}{2}$ in. grid. The contours are derived from observations at intervals of $\frac{1}{8}$ in.

In the region of the grid corresponding to figure 1, the diameter of the bars is very uniform and the spacing does not vary by more than 2%. The intersections are soldered but the solder is in no case visible in the projection seen by the wind, although the shape of the fillets varies slightly. There is no significant correlation between the intensities in figure 1 and the area of the grid openings; nevertheless, the pattern is a property of the grid, since when the grid is displaced sideways the pattern follows it, at least for small displacements.

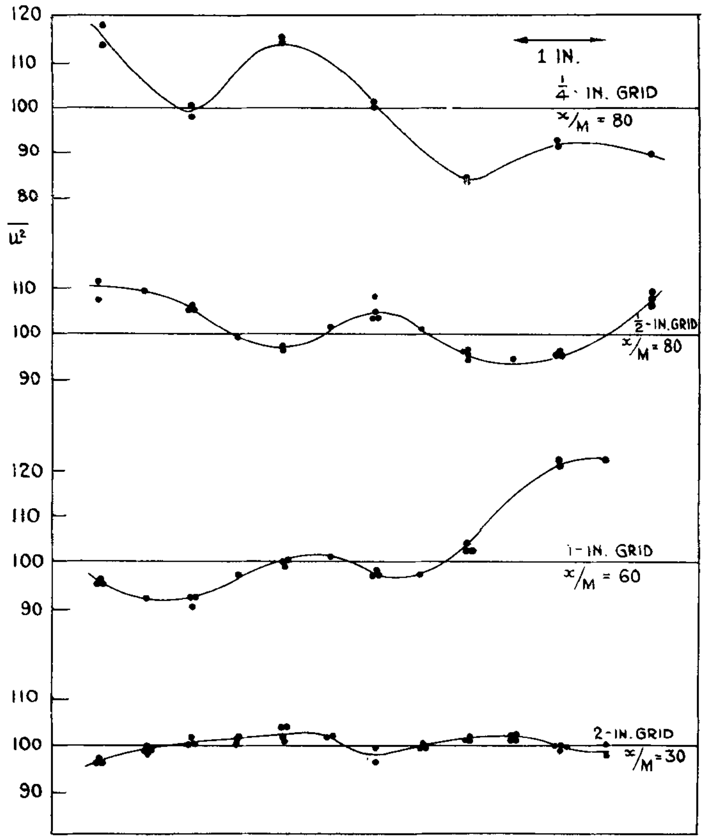

Figure 2. Variation of $\overline{u^2}$ (in arbitrary units) along a line normal to the mean flow with various grids.

Similar effects are found behind all the grids which we have examined. Figure 2 shows several examples of the variation of the intensity along lines normal to the mean stream. The larger meshes produce more uniform turbulence, although variations of as much as 6% are found behind the 2 in. grids.

These patterns are found to change only slowly with distance from the grid, although none has been traced closer than about thirty mesh-lengths from the grid. That such patterns, once formed, would be expected to persist for a long distance from the grid may be shown by the following rough calculation. If an excessively high intensity is being produced at a point in the plane of the grid, the subsequent spread of this high intensity in the lateral direction can be regarded as a diffusion problem, with the diffusion coefficient K given by the product of a representative length scale L, and a representative velocity $(\overline{u^2})^{\frac{1}{2}}$ of the turbulence. If these are constant, the spread of turbulent energy in the lateral plane is equivalent to heat conduction from a line source. The distribution of excess intensity will be like

$$\frac{1}{Kt}\, e^{-y^2/4Kt},$$

where t is time measured from the instant the fluid left the grid and y is distance normal to the mean streamline which passes through the source of high intensity. The half-width of the distribution is $2(Kt)^{\frac{1}{2}}$, which is equal to $2\{Lx(\overline{u^2})^{\frac{1}{2}}/U\}^{\frac{1}{2}}$. If we assume that $L/M = \frac{1}{2}$, and $(\overline{u^2})^{\frac{1}{2}}/U = 0{\cdot}014$ at $x/M = 50$, where M is the meshlength, then $2(Kt)^{\frac{1}{2}} = 0{\cdot}7$. If such sources are at least a meshlength apart, the turbulence has not had time, at 50 meshlengths from the grid, to become homogeneous through adjacent 'wakes' overlapping. It may also be noted that until adjacent 'wakes' do overlap appreciably, the quantity $1/Kt$ in the equation is decreasing as $x^{-\frac{1}{2}}$, so that the excess intensity is decreasing rather slowly along a streamline.

Inhomogeneity of the cross-stream components

The intensities of the cross-stream components of velocity have similar variations, but their spatial distributions are not obviously correlated with

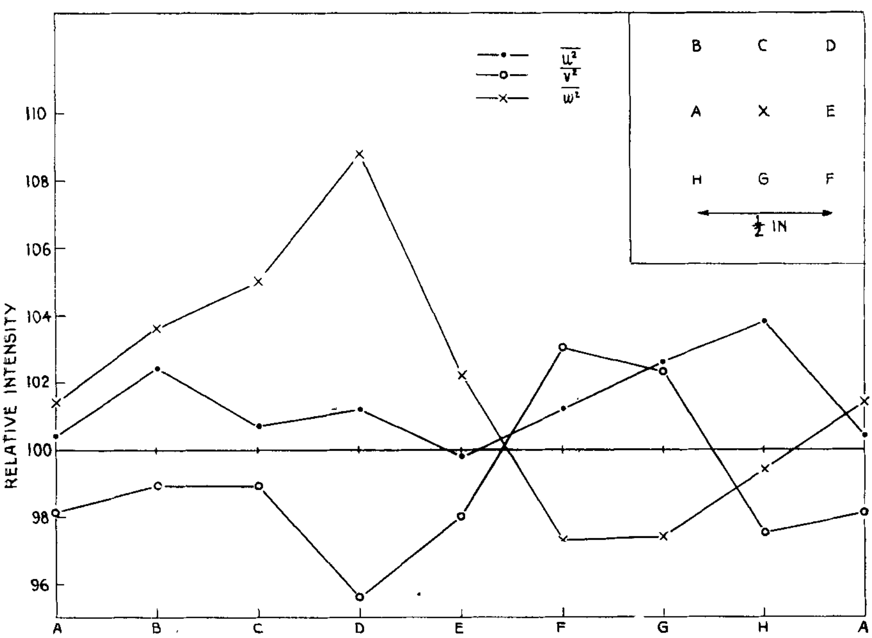

Figure 3. The distribution of the intensities of the three components of turbulent velocity near the point X in figure 1. The letters on the abscissa refer to points located with respect to X as shown in the inset. The intensity of each component is taken as 100 at X.

each other or with that of the intensity of the downstream component. An example from the $\frac{1}{2}$ in. grid is given in figure 3, which shows the distribution of intensity about the point X in figure 1. This point was in the region where $\overline{u^2}$ was most uniform, yet quite large variations were found in the other components. The obvious anticorrelation between $\overline{v^2}$ and $\overline{w^2}$ is probably not typical, but we have not measured all three components in enough cases to be sure. We do have a considerable number of cases where the intensities of the downstream and one cross-stream component have been recorded, and there is usually no obvious correlation between the two curves.

Absolute determination of intensity

It is clear from these observations that the ratios of the intensities of the different velocity components are not constant, so that the turbulence cannot be isotropic. We have made a series of determinations of $\overline{u^2}/U^2$, $\overline{v^2}/U^2$ and $\overline{v^2}/\overline{u^2}$, at 30 meshlengths downstream from one of the 2 in. grids at a point which will be referred to as reference point A.

For the determination of $\overline{u^2}/U^2$, a single hot wire normal to the stream was calibrated by varying the wind speed in the absence of a grid and observing the variation of heating current required to keep the wire at constant temperature. From this, $\overline{v^2}/U^2$ was obtained indirectly by using a single hot wire which could be operated in three positions, perpendicular to the mean stream and at 45° on each side of it. This arrangement gives signals proportional to the mean squares of u/U, $(u+v)/U$ and $(u-v)/U$, the ratios of the sensitivities being determined by the mean heating currents

U (cm/sec)	Component	Value	Remarks
630	$\overline{u^2}/U^2$	$4\cdot5 \times 10^{-4}$	Measured.
630	$\overline{v^2}/\overline{u^2}$	$0\cdot78 \pm 0\cdot02$	Measured.
630	$\overline{v^2}/U^2$	$3\cdot5 \times 10^{-4}$	Deduced from above figures.
1100	$\overline{u^2}/U^2$	$4\cdot3 \times 10^{-4}$	Measured.
1100	$\overline{v^2}/\overline{u^2}$	$0\cdot72 \pm 0\cdot01$	Measured.
1100	$\overline{v^2}/U^2$	$3\cdot1 \times 10^{-4}$	Deduced from above figures.
1100	$\overline{uv}/\overline{u^2}$	$-0\cdot105 \pm 0\cdot005$	Measured values of $\overline{uv}/\overline{u^2}$ varied between $-0\cdot08$ and $-0\cdot12$ over this lateral plane.

Table 1. Turbulent intensities at reference point A.

in the three positions. Accurate alignment with the mean stream can be effected by equalizing the mean heating currents in the two 45° positions, and hence values can be obtained for $\overline{u^2}/\overline{v^2}$ and $\overline{uv}/\overline{u^2}$. An attempt was also made to determine $\overline{v^2}/U^2$ directly, using an x-wire which was calibrated by rotating it in its own plane about an axis normal to a steady flow, thereby introducing a known v-component to the stream past the wire. This method did not give consistent results and was discarded.

In view of the inhomogeneity, the results (see table 1) are useful for calibration purposes only in this particular tunnel, but the value of $\overline{v^2}/\overline{u^2}$ is of more general interest because it is smaller than unity by about 25%. This is consistent with the values found by Corrsin and Wyatt, and is significantly different from unity since the maximum variation of $\overline{u^2}$ and $\overline{v^2}$ is about 3% from the mean in the case of the 2 in. grids.

Decay of turbulent energy

The decay of turbulent energy in the 'initial period' has been the subject of a number of experimental studies and much theoretical discussion (see Batchelor (1953)). We do not know how the rate of decay of energy along a streamline of the mean flow depends upon the lateral intensity

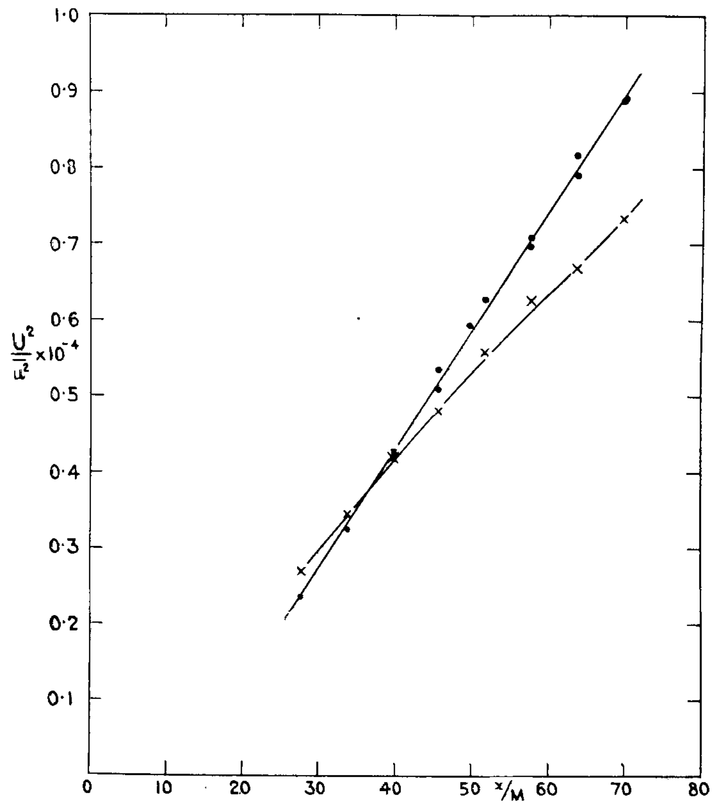

Figure 4. Decay curves taken at different lateral positions in the tunnel with the 1 in. grid. No great care was taken to see that the hot wire followed a mean streamline.

gradient, but it is certain that inconsistent results can easily be obtained through failure to make the intensity determinations for a decay curve along a single mean streamline. As an illustration of what can happen, we present in figure 4 two determinations of $U^2/\overline{u^2}$ as a function of x/M for a wind speed of 6·3 m/sec. The same grid was used, but the hot wire was moved down the tunnel at two different lateral positions without taking particular care to see that it followed a mean streamline. As long as the hot wire follows the same track, each of these curves is quite reproducible.

Other grids

Brief surveys were made of the turbulence behind two other types of grid. One consisted of a sheet of metal 0·02 in. thick in which $\frac{3}{8}$ in. holes had been punched at regular intervals in a hexagonal pattern leaving a blockage ratio of 0·48. The edges of the holes were very neat and the diameter did not vary by more than 0·001 in. The distance between the holes was uniform to within 0·4% in one direction but there were variations of 4% from the mean in the perpendicular direction. The intensity behind this grid was very irregular in both directions but slightly more so in the direction along which the hole spacing was not uniform. At one place the intensity changed by a factor of two in 2 in. The other grid was a single array of parallel rectangular prisms of section 3·7 cm × 0·4 cm, spaced 1·9 cm apart (Stewart & Townsend 1951). Here the variations over most of the tunnel section were within only 2 or 3%, although at one point the intensity rose to 15% above the mean.

Discussion

The possibility of appreciable inhomogeneity has not been considered when previous measurements of the properties of grid turbulence have been interpreted with the aid of the theory of isotropic turbulence, and it is necessary to consider whether or not appreciable errors may have occurred. In most measurements one of the independent variables has been distance downstream, and it can now be seen that unless care is taken to prevent it, the probe may wander from a mean streamline on which the turbulence perhaps has a high value of $\overline{u^2}$ to a mean streamline on which $\overline{u^2}$ is lower than the average value over a lateral plane. We believe that this is the chief cause of the unreliability of our decay curves, but it is also possible that there are real differences between the decay curves at different parts of the flow and that these contribute to the variations among the observations. It seems likely that these differences would be small where the lateral intensity gradient is not large. If this is so, then a significant determination of the rate of decay could be obtained by following a mean streamline. To follow a mean streamline with sufficient accuracy would not be difficult with the 2 in. and larger grids; with which the scale of the variations is correspondingly large; however, a very long wind tunnel is needed to give access to the interesting final period of decay with these grids.

Studies of the 'approach to isotropy' appear to be very unreliable if made in this kind of turbulence, as we have obtained curves of $\overline{v^2}/\overline{u^2}$ against distance from the grid which sometimes had a positive slope and sometimes a negative slope. Most such studies, however, have been conducted with turbulence which has deliberately been made highly anisotropic (Townsend 1950, 1954; Batchelor & Stewart 1950), and it is hard to estimate the effect of an initial inhomogeneity in the ratio $\overline{v^2}/\overline{u^2}$. In these cases any error is more likely to have arisen from the assumption, used in calibration of the hot wires, that $\overline{v^2} = \overline{u^2}$ in grid turbulence in the initial period of decay.

Similar comments could be made about functions of the velocity derivatives. In the one case in which $\overline{(\partial u/\partial x)^2}$ was measured, it was found to vary in a similar manner to $\overline{u^2}$, but it is not safe to conclude from this one observation that the spatial variations of the two quantities are always correlated.

Of the correlation functions, $f(r) = R_{11}(r, 0, 0)/\overline{u^2}$ is the one of most interest in isotropic turbulence. It is probably not much affected by spatial variations in the intensity unless the proportional distribution of energy among the various scales of turbulence also varies in the lateral direction. We have no information about this. Another correlation component which has often been measured is $g(r)$, one form of which is $R_{11}(0, r, 0)/\overline{u^2}$. The most closely related quantity which can be defined in a flow which is not homogeneous is the correlation coefficient $R_{11}(0, r, 0)/(\overline{u^2}\,\overline{u'^2})^{\frac{1}{2}}$, which in fact is what has actually been measured in most cases. ($\overline{u^2}$ and $\overline{u'^2}$ are the intensities at the positions of the two probes.) This is determined without obtaining the two intensities explicitly, and is independent of variations of intensity although it would be affected by variations of scale.

There are certain measurable properties of the turbulence which are likely to depend only on the gross geometry of the grid and which could be expected to hold for wind tunnels and grids other than those with which the measurements were made. One such quantity might be the mean, over many points in a lateral plane at a fixed distance from the grid, of the local time average of u^2. This combined space and time average might be an acceptable approximation to the intensity in the theoretical homogeneous turbulence. However, this mean is derived from a velocity distribution which may not be independent of the detailed initial conditions prevailing at the region of formation of the turbulence (detailed geometry of the intersections, level of turbulence in the incident stream, etc.), so higher moments might be very different from those of homogeneous turbulence even if defined in the above way.

For a proper understanding of the relation between real grid turbulence and truly homogeneous turbulence, it would be necessary at least to make a study of the decay along mean streamlines. Even more useful would be to find a way of making turbulence which is really homogeneous if not isotropic.

Cause of the inhomogeneity

Very little can be said about the cause of the phenomenon. It may be that the point of separation on the cylinders forming the grid is sensitive to very small changes in the geometry of the intersections, or to surface roughness, although this is not likely to be a factor with the grid made by punching holes in a sheet. One of the authors is at present conducting a study of the flow in the region immediately behind the grid and it is hoped that this may throw some light on the subject.

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